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# On the $\Delta\Delta$ component of the deuteron in the Nambu-Jona-Lasinio model of light nuclei

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**Abstract.** The probability $P(\Delta\Delta)$ to find the $\Delta\Delta$ component inside the deuteron, where $\Delta$ stands for the $\Delta(1232)$ resonance, is calculated in the Nambu-Jona-Lasinio model of light nuclei. We obtain $P(\Delta\Delta) = 0.3\%$. This prediction agrees well with the experimental estimate $P(\Delta\Delta) < 0.4\%$ at 90% of CL (D. Allasia *et al.*, Phys. Lett. B **174**, 450 (1986)).

**PACS.** 11.10.Ef Lagrangian and Hamiltonian approach – 13.75.Cs Nucleon-nucleon interactions (including antinucleons, deuterons, etc.) – 14.20.Dh Protons and neutrons – 21.30.Fe Forces in hadronic systems and effective interactions

## 1 Introduction

As has been stated in ref. [1], nowadays there is a consensus concerning the existence of non-nucleonic degrees of freedom in nuclei. The non-nucleonic degrees of freedom can be described either within QCD in terms of quarks and gluons [2] or in terms of mesons and nucleon resonances [3].

In this letter we investigate the non-nucleonic degrees of freedom in terms of the $\Delta(1232)$ resonance and calculate the contribution of the $\Delta\Delta$ component to the deuteron in the Nambu-Jona-Lasinio model of light nuclei or differently the nuclear Nambu-Jona-Lasinio (NNJL) model [4,5]. As has been shown in ref. [4] the NNJL model is motivated by QCD. The deuteron appears *in the nuclear phase of QCD* as a neutron-proton collective excitation, the Cooper np-pair, induced by a phenomenological local four-nucleon interaction. The NNJL model describes low-energy nuclear forces in terms of one-nucleon loop exchanges providing a minimal transfer of nucleon flavours from initial to final nuclear states and accounting for contributions of nucleon-loop anomalies which are completely determined by one-nucleon loop diagrams. The dominance of contributions of nucleon-loop anomalies to effective Lagrangians of low-energy nuclear interactions is justified in the large $N_{\rm C}$ expansion, where $N_{\rm C}$ is the number of quark colours [4]. As has been shown in ref. [5] the NNJL model describes well low-energy nuclear forces for electromagnetic and weak nuclear reactions with the deuteron of astrophysical interest such as the neutron-proton radiative capture $\rm n + p \to D + \gamma$, the solar proton burning $\rm p + p \to D + e^+ + \nu_e$, the pep-process $\rm p + e^- + p \to D + \nu_e$ and reactions of the disintegration of the deuteron by neutrinos and antineutrinos caused by charged $\rm \nu_e + D \to e^- + p + p, \bar{\nu}_e + D \to e^+ + n + n$ and neutral $\rm \nu_e(\bar{\nu}_e) + D \to \nu_e(\bar{\nu}_e) + n + p$ weak currents.

A phenomenological Lagrangian of the npD interaction is defined by [4]

$$\mathcal{L}_{\rm npD}(x) = -ig_{\rm V}[\bar{p}(x)\gamma^{\mu}n^c(x)\bar{n}(x)\gamma^{\mu}p^c(x)]D_{\mu}(x), \quad (1.1)$$

where $D_{\mu}(x)$, $n(x)$ and $p(x)$ are the interpolating fields of the deuteron, the neutron and the proton. The phenomenological coupling constant $g_{\rm V}$ is related to the electric quadrupole moment of the deuteron $Q_{\rm D} = 0.286\,{\rm fm}$: $g^2_{\rm V} = 2\pi^2 Q_{\rm D} M^2_{\rm N}$ [4], where $M_{\rm N} = 940\,{\rm MeV}$ is the nucleon mass. In the isotopically invariant form the phenomenological interaction equation (1.1) can be written as

$$\mathcal{L}_{\rm npD}(x) = g_{\rm V}\bar{N}(x)\gamma^{\mu}\tau_2 N^c(x)D_{\mu}(x), \quad (1.2)$$

where $\tau_2$ is the Pauli isotopical matrix and $N(x)$ is a doublet of a nucleon field with components $N(x) = (p(x), n(x))$, $N^c(x) = C\bar{N}^{\rm T}(x)$ and $\bar{N}^c(x) = N^{\rm T}(x)C$, where $C$ is a charge conjugation matrix and T is a transposition.

In the NNJL model [5] the $\Delta(1232)$ resonance is the Rarita-Schwinger field [6] $\Delta^a_{\mu}(x)$, the isotopical index $a$ runs over $a = 1, 2, 3$, having the following free La-

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grangian [7,8]:

$$\mathcal{L}^{\Delta}_{\rm kin}(x) = \bar{\Delta}^a_\mu(x)\Big[ -(i\gamma^\alpha\partial_\alpha - M_\Delta)g^{\mu\nu} + \frac{1}{4}\gamma^\mu\gamma^\beta(i\gamma^\alpha\partial_\alpha - M_\Delta)\gamma_\beta\gamma^\nu\Big]\Delta^a_\nu(x), \qquad (1.3)$$

where $M_\Delta = 1232\,\mathrm{MeV}$ is the mass of the $\Delta(1232)$ resonance field $\Delta^a_\mu(x)$. In terms of the eigenstates of the electric charge operator the fields $\Delta^a_\mu(x)$ are given by [7,8]

$$\Delta^1_\mu(x) = \frac{1}{\sqrt{2}}\begin{pmatrix}\Delta^{++}_\mu(x) - \Delta^0_\mu(x)/\sqrt{3}\\ \Delta^+_\mu(x)/\sqrt{3} - \Delta^-_\mu(x)\end{pmatrix},$$
$$\Delta^2_\mu(x) = \frac{i}{\sqrt{2}}\begin{pmatrix}\Delta^{++}_\mu(x) + \Delta^0_\mu(x)/\sqrt{3}\\ \Delta^+_\mu(x)/\sqrt{3} + \Delta^-_\mu(x)\end{pmatrix},$$
$$\Delta^3_\mu(x) = -\sqrt{\frac{2}{3}}\begin{pmatrix}\Delta^{+}_\mu(x)\\ \Delta^0_\mu(x)\end{pmatrix}. \qquad (1.4)$$

The fields $\Delta^a_\mu(x)$ obey the subsidiary constraints: $\partial^\mu\Delta^a_\mu(x) = \gamma^\mu\Delta^a_\mu(x) = 0$ [7–9]. The Green function of the free $\Delta$-field is determined by

$$\langle 0|\mathrm{T}(\Delta_\mu(x_1)\bar{\Delta}_\nu(x_2))|0\rangle = -iS_{\mu\nu}(x_1 - x_2). \qquad (1.5)$$

In the momentum representation $S_{\mu\nu}(x)$ reads [5–8]

$$S_{\mu\nu}(p) = \frac{1}{M_\Delta - \hat{p}}\bigg( -g_{\mu\nu} + \frac{1}{3}\gamma_\mu\gamma_\nu + \frac{1}{3}\frac{\gamma_\mu p_\nu - \gamma_\nu p_\mu}{M_\Delta} + \frac{2}{3}\frac{p_\mu p_\nu}{M^2_\Delta}\bigg). \qquad (1.6)$$

The most general form of the $\pi\mathrm{N}\Delta$ interaction compatible with the requirements of chiral symmetry reads [7]:

$$\begin{aligned}\mathcal{L}_{\pi\mathrm{N}\Delta}(x) &= \frac{g_{\pi\mathrm{N}\Delta}}{2M_\mathrm{N}}\bar{\Delta}^a_\omega(x)\Theta^{\omega\varphi}N(x)\partial_\varphi\pi^a(x) + \mathrm{h.c.}\\ &= \frac{g_{\pi\mathrm{N}\Delta}}{\sqrt{6}M_\mathrm{N}}\bigg[\frac{1}{\sqrt{2}}\bar{\Delta}^+_\omega(x)\Theta^{\omega\varphi}n(x)\partial_\varphi\pi^+(x)\\ &\quad -\frac{1}{\sqrt{2}}\bar{\Delta}^0_\omega(x)\Theta^{\omega\varphi}p(x)\partial_\varphi\pi^-(x)\\ &\quad -\bar{\Delta}^+_\omega(x)\Theta^{\omega\varphi}p(x)\partial_\varphi\pi^0(x)\\ &\quad -\bar{\Delta}^0_\omega(x)\Theta^{\omega\varphi}p(x)\partial_\varphi\pi^0(x) + \cdots\bigg],\end{aligned} \qquad (1.7)$$

where $\pi^a(x)$ is the pion field with the components $\pi^1(x) = (\pi^-(x) + \pi^+(x))/\sqrt{2}$, $\pi^2(x) = (\pi^-(x) - \pi^+(x))/i\sqrt{2}$ and $\pi^3(x) = \pi^0(x)$. The tensor $\Theta^{\omega\varphi}$ is given in ref. [7]: $\Theta^{\omega\varphi} = g^{\omega\varphi} - (Z + 1/2)\gamma^\omega\gamma^\varphi$, where the parameter $Z$ is arbitrary. The parameter $Z$ defines the $\pi\mathrm{N}\Delta$ coupling off-mass shell of the $\Delta(1232)$-resonance. There is no consensus on the exact value of $Z$. From the theoretical point of view $Z = 1/2$ is preferred [7]. Phenomenological studies give only the bound $|Z| \le 1/2$ [9]. The value of the coupling constant $g_{\pi\mathrm{N}\Delta}$ relative to the coupling constant $g_{\pi\mathrm{NN}}$ is $g_{\pi\mathrm{N}\Delta} = 2g_{\pi\mathrm{NN}}$ [10]. As has been shown in ref. [5] for the description of the experimental value of the cross-section for the neutron-proton radiative capture for thermal neutrons, the parameter $Z$ should be equal to $Z = 0.473$. This agrees with the experimental bound [9]. At $Z = 1/2$ we get the result agreeing with the experimental value of the cross-section for the neutron-proton radiative capture with accuracy about 3% [5].

For the subsequent calculations of the $\Delta\Delta$ component of the deuteron it is useful to have the Lagrangian of the $\pi\mathrm{N}\Delta$ interaction taken in the equivalent form

$$\mathcal{L}_{\pi\mathrm{N}\Delta}(x) = \frac{g_{\pi\mathrm{N}\Delta}}{2M_\mathrm{N}}\partial_\varphi\pi^a(x)\bar{N}^c(x) \times \Theta^{\varphi\omega}\Delta^a_\omega(x)^c + \mathrm{h.c.}, \qquad (1.8)$$

where $\Delta^a_\omega(x)^c = C\bar{\Delta}^a_\omega(x)^\mathrm{T}$. Now we can proceed to the evaluation of the $\Delta\Delta$ component of the deuteron.

## 2 Effective $\Delta\Delta$D interaction

In the NNJL model we can understand the existence of the $\Delta\Delta$ component of the deuteron in terms of the coupling constants of the effective $\Delta\Delta$D interaction.

In order to evaluate the Lagrangian of the effective $\Delta\Delta$D interaction $\mathcal{L}^{\Delta\Delta\mathrm{D}}_{\rm eff}(x)$, we have to obtain, first, the effective Lagrangian of the transition $\mathrm{N}+\mathrm{N} \to \Delta + \Delta$. We define this effective Lagrangian in the one-pion exchange approximation [5,11]

$$\begin{aligned}\int \mathrm{d}^4x\,\mathcal{L}^{\mathrm{NN}\to\Delta\Delta}_{\rm eff}(x) =\\ -\frac{g^2_{\pi\mathrm{N}\Delta}}{8M^2_\mathrm{N}}\int\!\!\int \mathrm{d}^4x_1\,\mathrm{d}^4x_2\big[\bar{\Delta}^a_\alpha(x_1)\Theta^{\alpha\beta}N(x_1)\big]\\ \times\frac{\partial}{\partial x^\beta_1}\frac{\partial}{\partial x^\varphi_1}\big[\delta^{ab}\Delta(x_1 - x_2)\big]\big[\bar{N}^c(x_2)\Theta^{\varphi\omega}\Delta^b_\omega(x_2)^c\big],\end{aligned} \qquad (2.1)$$

where $\Delta(x_1 - x_2)$ is the Green function of $\pi$-mesons. In terms of the Lagrangians of the npD interaction and the $\mathrm{N}+\mathrm{N} \to \Delta + \Delta$ transition the Lagrangian of the effective $\Delta\Delta$D interaction can be defined by

$$\begin{aligned}\int \mathrm{d}^4x\,\mathcal{L}^{\Delta\Delta\mathrm{D}}_{\rm eff}(x) =\\ -ig_\mathrm{V}\frac{g^2_{\pi\mathrm{N}\Delta}}{4M^2_\mathrm{N}}\int \mathrm{d}^4x\,\mathrm{d}^4x_1\mathrm{d}^4x_2D_\mu(x)\\ \times\big[\bar{\Delta}^a_\alpha(x_1)\Theta^{\alpha\beta}S_\mathrm{F}(x - x_1)\gamma^\mu\tau_2S^c_\mathrm{F}(x - x_2)\\ \times\Theta^{\varphi\omega}\Delta^a_\omega(x_2)^c\big]\frac{\partial}{\partial x^\beta_1}\frac{\partial}{\partial x^\varphi_1}\Delta(x_1 - x_2),\end{aligned} \qquad (2.2)$$

where $S_\mathrm{F}(x - x_1)$ and $S^c_\mathrm{F}(x - x_2)$ are the Green functions of the free nucleon and anti-nucleon fields, respectively.

Such a definition of the contribution of the $\Delta\Delta$ component to the deuteron is in agreement with that given

$$\begin{aligned}\mathcal{L}_{\rm eff}^{\Delta\Delta\rm D}(x) &= g_{\Delta\Delta\rm D}\big[\bar{\Delta}^a_\alpha(x)\Theta^{\alpha\beta}\gamma^\mu\Theta_\beta{}^\omega\tau_2\Delta^a_\omega(x)^c\big]D_\mu(x) + \bar{g}_{\Delta\Delta\rm D}\big[\bar{\Delta}^a_\alpha(x)(\Theta^{\alpha\beta}\gamma_\beta\Theta^{\mu\omega} + \Theta^{\alpha\mu}\gamma_\varphi\Theta^{\varphi\omega})\tau_2\Delta^a_\omega(x)^c\big]D_\mu(x)\\ &= -ig_{\Delta\Delta\rm D}\big[\bar{\Delta}^-_\alpha(x)\Theta^{\alpha\beta}\gamma^\mu\Theta_\beta{}^\omega\Delta^{++}_\omega(x)^c - \bar{\Delta}^{++}_\alpha(x)\Theta^{\alpha\beta}\gamma^\mu\Theta_\beta{}^\omega\Delta^-_\omega(x)^c\\ &\quad + \bar{\Delta}^+_\alpha(x)\Theta^{\alpha\beta}\gamma^\mu\Theta_\beta{}^\omega\Delta^0_\omega(x)^c - \bar{\Delta}^0_\alpha(x)\Theta^{\alpha\beta}\gamma^\mu\Theta_\beta{}^\omega\Delta^+_\omega(x)^c\big]D_\mu(x)\\ &\quad - i\bar{g}_{\Delta\Delta\rm D}\big[\bar{\Delta}^-_\alpha(x)\big(\Theta^{\alpha\beta}\gamma_\beta\Theta^{\mu\omega} + \Theta^{\alpha\mu}\gamma_\varphi\Theta^{\varphi\omega}\big)\Delta^{++}_\omega(x)^c - \bar{\Delta}^{++}_\alpha(x)\big(\Theta^{\alpha\beta}\gamma_\beta\Theta^{\mu\omega} + \Theta^{\alpha\mu}\gamma_\varphi\Theta^{\varphi\omega}\big)\Delta^-_\omega(x)^c\\ &\quad + \bar{\Delta}^+_\alpha(x)\big(\Theta^{\alpha\beta}\gamma_\beta\Theta^{\mu\omega} + \Theta^{\alpha\mu}\gamma_\varphi\Theta^{\varphi\omega}\big)\Delta^0_\omega(x)^c - \bar{\Delta}^0_\alpha(x)\big(\Theta^{\alpha\beta}\gamma_\beta\Theta^{\mu\omega} + \Theta^{\alpha\mu}\gamma_\varphi\Theta^{\varphi\omega}\big)\Delta^+_\omega(x)^c\big], \end{aligned} \tag{2.7}$$

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by Niephaus *et al.* [12] in the potential model approach (PMA).

For the evaluation of the effective Lagrangian $\mathcal{L}_{\rm eff}^{\Delta\Delta\rm D}(x)$ we would follow the large $N_{\rm C}$ expansion approach to non-perturbative QCD [4]. In the large $N_{\rm C}$ approach to non-perturbative QCD with $SU(N_{\rm C})$ gauge group at $N_{\rm C}\to\infty$ the nucleon mass is proportional to the number of quark colour degrees of freedom, $M_{\rm N}\sim N_{\rm C}$ [13]. It is well-known that for the evaluation of effective Lagrangians all momenta of interacting particles should be kept off-mass shell. This implies that at leading order in the large $N_{\rm C}$ expansion corresponding to the $1/M_{\rm N}$ expansion of the momentum integral defining the effective Lagrangian $\mathcal{L}_{\rm eff}^{\Delta\Delta\rm D}(x)$ one can neglect the momenta of interacting particles with respect to the mass of virtual nucleons. As a result the effective Lagrangian $\mathcal{L}_{\rm eff}^{\Delta\Delta\rm D}(x)$ reduces to the local form and reads

$$\mathcal{L}_{\rm eff}^{\Delta\Delta\rm D}(x) = \frac{g_{\rm V}}{16\pi^2}\frac{g^2_{\pi\rm N\Delta}}{4M^2_{\rm N}}\big[\bar{\Delta}^a_\alpha(x)\Theta^{\alpha\mu\omega}\tau_2\Delta^a_\omega(x)^c\big]\,D_\mu(x), \tag{2.3}$$

where the structure function $\Theta^{\alpha\mu\omega}$ is given by the momentum integral

$$\Theta^{\alpha\mu\omega} = \int\frac{{\rm d}^4k}{\pi^2 i}\,\frac{1}{M^2_\pi - k^2}\times\Theta^{\alpha\beta}k_\beta\frac{1}{M_{\rm N}-\hat{k}}\gamma^\mu\frac{1}{M_{\rm N}+\hat{k}}k_\varphi\Theta^{\varphi\omega}. \tag{2.4}$$

Integrating over $k$ we obtain

$$\begin{aligned}\Theta^{\alpha\mu\omega} &= \frac{1}{3}\Big[I_1(M_{\rm N}) - \frac{5}{2}M^2_{\rm N}I_2(M_{\rm N})\Big]\Theta^{\alpha\beta}\gamma^\mu\Theta_\beta{}^\omega\\ &\quad - \frac{1}{12}\Big[I_1(M_{\rm N}) - M^2_{\rm N}I_2(M_{\rm N})\Big]\\ &\quad\times(\Theta^{\alpha\beta}\gamma_\beta\Theta^{\mu\omega} + \Theta^{\alpha\mu}\gamma_\varphi\Theta^{\varphi\omega}),\end{aligned} \tag{2.5}$$

where the quadratically, $I_1(M_{\rm N})$, and logarithmically, $I_2(M_{\rm N})$, divergent integrals are determined by [4]

$$\begin{aligned} I_1(M_{\rm N}) &= \int\frac{{\rm d}^4k}{\pi^2 i}\,\frac{1}{M^2_{\rm N}-k^2} = 2\Bigg[\Lambda\sqrt{M^2_{\rm N}+\Lambda^2} - M^2_{\rm N}\ln\Bigg(\frac{\Lambda}{M_{\rm N}} + \sqrt{1+\frac{\Lambda^2}{M^2_{\rm N}}}\Bigg)\Bigg],\\ I_2(M_{\rm N}) &= \int\frac{{\rm d}^4k}{\pi^2 i}\,\frac{1}{(M^2_{\rm N}-k^2)^2} = 2\Bigg[\ln\Bigg(\frac{\Lambda}{M_{\rm N}} + \sqrt{1+\frac{\Lambda^2}{M^2_{\rm N}}}\Bigg) - \frac{\Lambda}{\sqrt{M^2_{\rm N}+\Lambda^2}}\Bigg]. \end{aligned} \tag{2.6}$$

The cut-off $\Lambda$ restricts from above 3-momenta of fluctuating nucleon fields. Since we have no closed nucleon loops, the cut-off $\Lambda$ cannot be determined by the scale of the deuteron size $r_{\rm D}\sim 1/\Lambda_{\rm D}$ [4]. The natural value of $\Lambda$ is the scale of the Compton wavelength of the nucleon $\lambda\!\!\!^{-}_{\rm N} = 1/M_{\rm N} = 0.21\,{\rm fm}$, *i.e.* $\Lambda = M_{\rm N}$.

We obtain the Lagrangian $\mathcal{L}_{\rm eff}^{\Delta\Delta\rm D}(x)$ of the effective $\Delta\Delta$D interaction in the form

*see equation* (2.7) *above*

where the effective coupling constants $g_{\Delta\Delta\rm D}$ and $\bar{g}_{\Delta\Delta\rm D}$ read

$$\begin{aligned} g_{\Delta\Delta\rm D} &= g_{\rm V}\frac{7g^2_{\pi\rm N\Delta}}{384\pi^2}\Bigg[\frac{\Lambda}{\sqrt{M^2_{\rm N}+\Lambda^2}}\Bigg(1+\frac{2}{7}\frac{\Lambda^2}{M^2_{\rm N}}\Bigg) - \ln\Bigg(\frac{\Lambda}{M_{\rm N}} + \sqrt{1+\frac{\Lambda^2}{M^2_{\rm N}}}\Bigg)\Bigg],\\ \bar{g}_{\Delta\Delta\rm D} &= -g_{\rm V}\frac{g^2_{\pi\rm N\Delta}}{192\pi^2}\Bigg[\frac{\Lambda}{\sqrt{M^2_{\rm N}+\Lambda^2}}\Bigg(1+\frac{1}{2}\frac{\Lambda^2}{M^2_{\rm N}}\Bigg) - \ln\Bigg(\frac{\Lambda}{M_{\rm N}} + \sqrt{1+\frac{\Lambda^2}{M^2_{\rm N}}}\Bigg)\Bigg]. \end{aligned} \tag{2.8}$$

On-mass shell of the $\Delta(1232)$ resonance, *i.e.* in the case of the PMA [1,12], the contribution of the parameter $Z$ vanishes and the effective $\Delta\Delta$D interaction acquires the

$$\mathrm{d}\Gamma\big(\mathrm{D}(P) \to \Delta(p_1)\Delta(p_2)\big) = 8g^2_{\Delta\Delta\mathrm{D}}\frac{\mathrm{d}\Phi_{\Delta\Delta}(p_1,p_2)}{6\sqrt{s}}\bigg(-g_{\mu\nu} + \frac{P_\mu P_\nu}{s}\bigg)$$
$$\times\,\mathrm{tr}\bigg\{(M_\Delta + \hat{p}_1)\bigg(-g_{\alpha\beta} + \frac{1}{3}\gamma_\alpha\gamma_\beta + \frac{1}{3}\,\frac{\gamma_\alpha p_{1\beta} - \gamma_\beta p_{1\alpha}}{M_\Delta} + \frac{2}{3}\,\frac{p_{1\alpha}p_{1\beta}}{M^2_\Delta}\bigg)\gamma^\mu$$
$$\times\bigg(-g^{\alpha\beta} + \frac{1}{3}\gamma^\beta\gamma^\alpha + \frac{1}{3}\,\frac{\gamma^\beta p_2^\alpha - \gamma^\alpha p_2^\beta}{M_\Delta} + \frac{2}{3}\,\frac{p_2^\beta p_2^\alpha}{M^2_\Delta}\bigg)(-M_\Delta + \hat{p}_2)\gamma^\nu\bigg\},$$
$$\mathrm{d}\Gamma\big(\mathrm{D}(P) \to \mathrm{n}(p_1)\mathrm{p}(p_2)\big) = 4g^2_{\mathrm{V}}\frac{\mathrm{d}\Phi_{\mathrm{np}}(p_1,p_2)}{6\sqrt{s}}\bigg(-g_{\mu\nu} + \frac{P_\mu P_\nu}{s}\bigg)\,\mathrm{tr}\big\{(M_{\mathrm{N}} + \hat{p}_1)\gamma^\mu(-M_{\mathrm{N}} + \hat{p}_2)\gamma^\nu\big\}. \tag{2.11}$$

form

$$\mathcal{L}^{\Delta\Delta\mathrm{D}}_{\mathrm{eff}}(x) = g_{\Delta\Delta\mathrm{D}}g^{\alpha\beta}\big[\bar{\Delta}^a_\alpha(x)\gamma^\mu\tau_2\Delta^a_\beta(x)^c\big]D_\mu(x)$$
$$= -ig_{\Delta\Delta\mathrm{D}}g^{\alpha\beta}\big[\bar{\Delta}^-_\alpha(x)\gamma^\mu\Delta^{++}_\beta(x)^c - \bar{\Delta}^{++}_\alpha(x)\gamma^\mu\Delta^-_\beta(x)^c + \bar{\Delta}^+_\alpha(x)\gamma^\mu\Delta^0_\beta(x)^c - \bar{\Delta}^0_\alpha(x)\gamma^\mu\Delta^+_\beta(x)^c\big]D_\mu(x). \tag{2.9}$$

We determine the total probability $P(\Delta\Delta)$ to find the $\Delta\Delta$ component inside the deuteron as follows:

$$P(\Delta\Delta) = \frac{\mathrm{d}\Gamma(\mathrm{D}\to\Delta\Delta)}{\mathrm{d}\Gamma(\mathrm{D}\to\mathrm{np})}, \tag{2.10}$$

where $\mathrm{d}\Gamma(\mathrm{D}\to\Delta\Delta)$ and $\mathrm{d}\Gamma(\mathrm{D}\to\mathrm{np})$ are the differential rates of the transitions $\mathrm{D}\to\Delta+\Delta$ and $\mathrm{D}\to\mathrm{n}+\mathrm{p}$, respectively, defined by

*see equation* (2.11) *above*

We have denoted as $P = p_1 + p_2$ and $P^2 = s$ the 4-momentum and the invariant squared mass of the deuteron, respectively. Then, $\mathrm{d}\Phi_{\Delta\Delta}(p_1,p_2)$ and $\mathrm{d}\Phi_{\mathrm{np}}(p_1,p_2)$ are the phase volumes of the $\Delta\Delta$ and np states. The two-particle phase volume is equal to

$$\mathrm{d}\Phi(p_1,p_2) = (2\pi)^4(P - p_1 - p_2) \times \frac{\mathrm{d}^3p_1}{(2\pi)^3 2E_1}\frac{\mathrm{d}^3p_2}{(2\pi)^3 2E_2}. \tag{2.12}$$

At leading order in the large $N_{\mathrm{C}}$ expansion, when we can neglect the mass difference between the $\Delta(1232)$ resonance and the nucleon, the phase volumes $\mathrm{d}\Phi_{\Delta\Delta}(p_1,p_2)$ and $\mathrm{d}\Phi_{\mathrm{np}}(p_1,p_2)$ are equal

$$\mathrm{d}\Phi_{\Delta\Delta}(p_1,p_2) = \mathrm{d}\Phi_{\mathrm{np}}(p_1,p_2) = \mathrm{d}\Phi(p_1,p_2). \tag{2.13}$$

The differential rates $\mathrm{d}\Gamma(\mathrm{D}(P)\to\Delta(p_1)\Delta(p_2))$ and $\mathrm{d}\Gamma(\mathrm{D}(P)\to\mathrm{n}(p_1)\mathrm{p}(p_2))$ calculated at leading order in the large $N_{\mathrm{C}}$ expansion are given by

$$\mathrm{d}\Gamma(\mathrm{D}(P)\to\Delta(p_1)\Delta(p_2)) = \frac{10}{9}\times 8\times g^2_{\Delta\Delta\mathrm{D}}\times\sqrt{s}\,\mathrm{d}\Phi(p_1,p_2),$$
$$\mathrm{d}\Gamma(\mathrm{D}(P)\to\mathrm{n}(p_1)\mathrm{p}(p_2)) = 4\times g^2_{\mathrm{V}}\times\sqrt{s}\,\mathrm{d}\Phi(p_1,p_2). \tag{2.14}$$

Hence, the probability $P(\Delta\Delta)$ to find the $\Delta\Delta$ component inside the deuteron amounts to

$$P(\Delta\Delta) = \frac{10}{9}\times\frac{2g^2_{\Delta\Delta\mathrm{D}}}{g^2_{\mathrm{V}}} = 0.3\%, \tag{2.15}$$

where the numerical value is obtained at $\Lambda = M_{\mathrm{N}}$.

Our theoretical prediction agrees well with recent experimental estimate of the upper limit $P(\Delta\Delta) < 0.4\%$ at 90% of CL [14] quoted by Dymarz and Khanna [1].

## 3 Conclusion

The theoretical estimate of the contribution of the $\Delta\Delta$ component to the deuteron obtained in the NNJL model agrees well with the experimental upper limit. Indeed, for the $\Delta(1232)$ resonance on-mass shell [1,12] we predict $P(\Delta\Delta) = 0.3\%$, whereas experimentally $P(\Delta\Delta)$ is restricted by $P(\Delta\Delta) < 0.4\%$ at 90% of CL [14].

Off-mass shell of the $\Delta(1232)$-resonance, where the parameter $Z$ should contribute, our prediction for $P(\Delta\Delta)$ can be changed, of course. Moreover, due to $Z$ dependence, the contributions of the $\Delta\Delta$ component to amplitudes of different low-energy nuclear reactions and physical quantities could differ from each other. However, we would like to emphasize that in the NNJL model by using the effective $\Delta\Delta$D interaction determined by eq. (2.7) one can calculate the contribution of the $\Delta\Delta$ component of the deuteron to the amplitude of any low-energy nuclear reaction with the deuteron in the initial or final state.

In our approach we do not distinguish contributions of the $\Delta\Delta$-pair with a definite orbital momentum ${}^3S_1^{\Delta\Delta}$, ${}^3D_1^{\Delta\Delta}$ and so on to the effective $\Delta\Delta$D interaction eq. (2.7). The obtained value of the probability $P(\Delta\Delta)$ should be considered as a sum of all possible states with a certain orbital momentum.

Our prediction $P(\Delta\Delta) = 0.3\%$ agrees reasonably well with the result obtained by Dymarz and Khanna in the PMA [1]: $P(\Delta\Delta) \simeq 0.4 \div 0.5\%$. Unlike our approach Dymarz and Khanna have given a percentage of the probabilities of different states ${}^3S_1^{\Delta\Delta}$, ${}^3D_1^{\Delta\Delta}$ and so to the wave function of the deuteron. In our approach the deuteron couples to itself and other particles through the one-baryon loop exchanges. The effective Lagrangian $\mathcal{L}_{\rm eff}^{\Delta\Delta\rm D}(x)$ of the $\Delta\Delta$D interaction given by eq. (2.7) defines completely the contribution of the $\Delta\Delta$ intermediate states to baryon-loop exchanges. The decomposition of the effective $\Delta\Delta$D interaction in terms of the $\Delta\Delta$ states with a certain orbital momentum should violate Lorentz invariance for the evaluation of the contribution of every state to either the amplitude of a low-energy nuclear reaction or a low-energy physical quantity. In the NNJL model this can lead to incorrect results. The relativistically covariant procedure of the decomposition of the interactions like the $\Delta\Delta$D one in terms of the states with a certain orbital momenta is now in progress in the NNJL model. However, the smallness of the contribution of the $\Delta\Delta$ component to the deuteron obtained in the NNJL model makes such a decomposition applied to the $\Delta\Delta$D interaction meaningless to some extent due to impossibility to measure the terms separately.

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